Physico-mathematical Sciences

Applied mathematics

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**WORKBOOK**

**HIGHER ENGINEERING MATHEMATICS**

**CHAPTER “COMPLEX NUMBERS” (part 2)**

This paper the author`s translation of the methodical paper [1] and is the continuation of publication [2].

**1.2 The trigonometric (polar, goniometric) form**

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Suppose  then 

Thus 

Thus the modulus of the product is the product of the moduli i.e.,  and the argument of the product is the sum of the arguments, i.e.  or 

**1.3 The Euler Formula**

The Euler Formula states:



Due to the Euler Formula the definition of the exponential of  a real number can be generalized to the exponential of a complex number 



In a special case, where *z* is a real number (that is ), this formula gives the desired result:



One can easily prove that the exponential of complex numbers have the same properties as the exponential of real numbers. For instance,

 .

The Euler Formula gives the following representation of a complex number in the polar form:

 .

**Example**



**1.4 Trigonometric applications for the Euler Formula**

All trigonometric identities can be easily derived by making use of **t**he Euler Formula. The following examples illustrate typical techniques.

1. The identities





follow from the Euler Formula by adding and subtracting of the equalities

 and  .

2. Let us square both sides of equality



On the other hand . Thus

 and 

3. Consider the product



On the other hand  (\*)  (\*\*)

Comparing (\*) with (\*\*) we conclude that





Regarding multiplication of complex numbers written in trigonometric form we have the following theorem:

**De Moiver`s formula:**

****

We put it into late use in the very important problem.

**Division**

 is defined as the inverse operation of multiplication. In practice  is obtained by multiplying the numerator or denominator by **,** the conjugate of  ***z*2**: .

Thus  .

Note that 

In trigonometric (polar) form



**Example:** represent in trigonometric (polar) form complex numbers    

Solution:

****

****

****

**Example:** represent in trigonometric (polar) form complex numbers    and find complex number 

Solution:

  

;

  

;

  

.

Thus



and

.

**Example:** Express in the algebraical form  and find its modulus

1.  2) 

Solution:

1)modulus of ==

2)

==

modulus =

**Example:** Solve the equation  given that  and  are real.

Solution:



   and

**

If  then  Since  real,  is not possible. Thus 

Assume  then  or 

Eliminating  from **  

Further  

Since  is real,  Then  or 

Thus   are the solutions.

In polar form  .

Thus the modulus of the quotient of the moduli i.e.,  and the argument of the difference between the arguments i.e.,

 or 

List of reference

1. Бажанов, Н.Н. Учебно-методическое пособие на модульной основе с диагностико-квалиметрическим обеспечением по курсу высшая математика/Н.Н.Бажанов. - Изд-во Технологического института ЮФУ, 2010. – 40 с.
2. Bazhanov N.N. Workbook higher engineering mathematics chapter “Complex numbers” (part 1) // URL: <http://conferences.neasmo.org.ua/ru>, 2015.